Microphone arrays around rigid spheres for spatial recording and holography

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Introduction

This paper presents a description of open microphone arrays enclosing a spherical scatterer of variable size. Both open and closed spherical array designs have been presented in recent literature, each exhibiting their own strengths and weaknesses. The performance of every individual design depends on factors including number and placement of capsules as well as the diameter and physical construction of the array. As frequency range and spatial resolution impose conflicting demands, this survey intends to improve spherical microphone arrays by carefully weighting their design parameters.

Spherical Harmonics Decomposition

A continuous sound pressure or particle velocity distribution measured on a spherical surface can be represented by a superposition of spherical harmonics. These harmonics form an orthogonal system and are the solutions of the angular part of the Laplace equation in spherical coordinates. The decomposition of a distribution $x(\boldsymbol{\theta})$ into an infinite number of harmonics is done by spatial Fourier transform, using the spherical harmonics $Y_n^m(\boldsymbol{\theta})$ as the transform kernel [1].

$$SHT_{nm}\{x(\boldsymbol{\theta})\} = \chi_n^m = \int_{\mathbb{S}^2} x(\boldsymbol{\theta})Y_n^m(\boldsymbol{\theta})\, d\boldsymbol{\theta} \qquad (1)$$

The spectrum χ_n^m represents the angular components of the distribution. When this distribution is sampled at L discrete points, the distribution can only be decomposed into a finite amount of harmonics, leading to limited precision.

$$DSHT_N\{\boldsymbol{x}_L\} = \boldsymbol{\chi}_N = \boldsymbol{Y}_N^{-1}\,\boldsymbol{x}_L \qquad (2)$$

Several grids for spherical surfaces with varying orders and sampling accuracy have been developed in the scientific community.

Physical Layouts and Holography Filters

To relate the transformed microphone signals χ_n^m to sound pressure or sound particle velocity, the type of microphone has to be taken into account. Let's assume the entire distribution $x(\boldsymbol{\theta})$ to be known on a surface of radius r.

Omnidirectional Microphones

Spherical microphone arrays with pressure microphones arranged in an open sphere do not provide information about sound particle velocity. The microphone signals
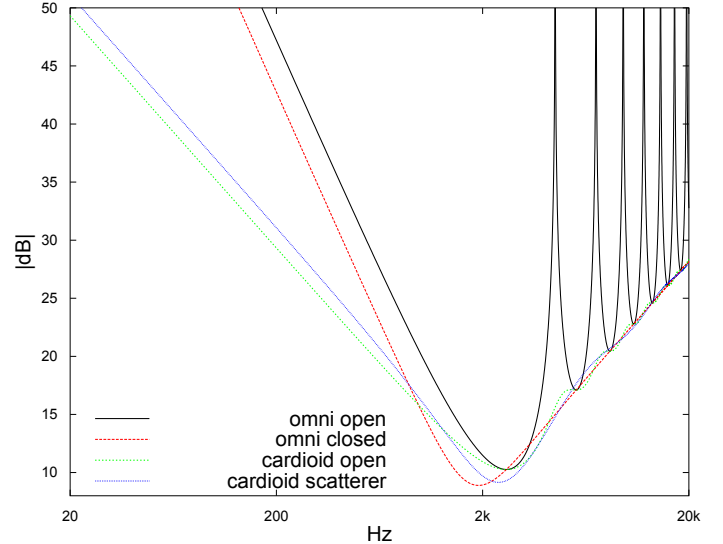

Figure 1: Filter magnitude for pressure and cardioid microphones around open and closed spheres and around a scatterer of radius $r_k = 5mm$. Order $N = 2$ and microphone radius $r = 70mm$

correspond to the sound pressure, the spherical harmonics representation of which is given in [2] as

$$\chi_n^m(kr) = \psi_n^m(kr) = b_{nm}j_n(kr) \qquad (3)$$

It is desirable to extract the wave spectral coefficients b_{nm} for holography. The division by the spherical Bessel function $j_n(kr)$, which is zero for certain values kr, is undefined. As is shown in Figure 1 "omni open" the magnitude of this division represents a holography filter. The placement of omnidirectional capsules flush mounted on a sound-hard sphere effectively forces the particle velocity to become zero on the surface and is thus no longer unknown. With this property the microphone signals become [1]

$$\chi_n^m(kr) = \psi_n^m(kr) = b_{nm}\left[j_n(kr) - \frac{j_n'(kr)}{h_n'(kr)}h_n(kr)\right] \quad (4)$$

with $h_n(x)$ being the spherical Hankel function of the second kind $h_n^{(2)}(x)$, and $'$ denoting the first derivative with regard to time. This approach results in a filter curve without singularities but nevertheless steep magnitude for low frequencies, as shown in Figure 1 "omni closed". In most physical microphone arrays the capsules, their mounting and the wiring will form a scattering obstacle of some kind. This object may not exhibit entirely reflective properties. A measurement of its acoustic impedance, preferably in the spherical harmonics domain, has to be taken.
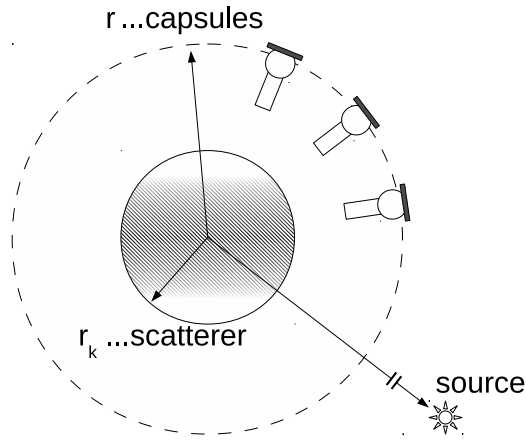
Figure 2: A cocentric layout with microphones at radius r around a scatterer of radius r_k

Cardioid Microphones

Microphone arrays built with cardioid microphones sense pressure and particle velocity. For such capsules arranged in an open sphere the microphone signals are a combination of sound pressure and particle velocity [3].

$$\chi_n^m(kr) = b_{nm} \left[j_n(kr) - \mathrm{i}j_n'(kr) \right] \qquad (5)$$

The holography filters have a lower magnitude at low frequencies, thus increasing the usable signal-to-noise ratio of the system, and exhibit the lowest requirements of all layouts discussed herein, as seen from Figure 1 "cardioid open". The same approach, but with the microphones placed around a scattering sphere of radius r_k as shown in Figure 2, introduces the reflected components into the microphone signal spectrum

$$\chi_n^m(kr) = \qquad (6)$$
$$b_{nm} \left[j_n(kr) - \mathrm{i}j_n'(kr) + (\mathrm{i}h_n'(kr) - h_n(kr)) \frac{j_n'(kr_k)}{h_n'(kr_k)} \right]$$

This filter magnitude, Figure 1 "cardioid open", now has a slightly worse behavior, but is still superior to omnidirectional layouts. However the directivity of real-world cardioid microphones varies with frequency. This behavior can be approximated with a modified version of equation (5) in which scaling factors weigh the pressure and velocitiy components independently. At low frequencies the zeroth order pressure component will be predominant. A more advanced approach includes transfer function measurements of a capsule for all angles. The spherical harmonics-transformed transfer functions can then be integrated into equation (6) by means of spherical convolution.

Improvements

For cardioid microphones around an open sphere the filter magnitude can be regarded as a highpass filter with 6dB per octave and a lowpass filter with $(N-1)$6dB per octave. Doubling the microphone radius shifts the two filters' common corner frequency down by one octave. Two strategies exist to work around the high gain requirements. One is to build a dual- or multi-sphere array,

using the outer array to capture lower frequencies. This allows decomposition of low and high frequencies at the same order N at the expense of more microphones physical dimensions of the array [4][3]. The other approach is to use spherical harmonics decomposition at different orders in different frequency bands [5]. Both strategies lose angular resolution at low frequencies by limiting the order N or increasing the space between sampling points. Spatial aliasing is the consequence of a limited number of microphones and hence spherical harmonics. Aliasing is most prominent for high frequencies. It increases as the spacing between microphones gets bigger, for example by enlarging the radius. An error measure for the overall accuracy of microphone array holography has been suggested in [6]. This holographic error is a scalar measure comparing an analytic far-field source with its possibly aliased and distorted replica as resulted from microphone array holography. The influence of imperfections such as gain errors and physical misalignment can be considered as well. This error is defined as a magnitude in dB and allows to define an upper boundary for artefacts which in turn defines the upper limit frequency for the array. It can be shown that doubling the array radius lowers the upper limit frequency for acoustic holography by one octave.

Conclusion

Microphone array design poses an exciting challenge in the consideration of its layout, construction of the hardware, microphone choice and algorithm development. The tradeoff at hand is about keeping a certain signal-to-noise ratio while preventing spatial aliasing. The filters presented for acoustic holography raise the noise floor of any real array implementation. It is important to keep their magnitudes as low as possible, which can be achieved by using cardioid microphones around an open sphere, by multi-sphere cocentric geometries, and by applying multiple decomposition orders in different frequency bands. A combination of these three strategies is desirable and subject to current research.

References

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