

SPIN QUARTETS. SONIFICATION OF THE XY MODEL.

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ABSTRACT

We present an intuitive sonification of data from a statistical physics model, the XY-spin model. Topological structures (anti-/vortices) are hidden to the eye by random spin movement. The behavior of the vortices changes by crossing a phase transition as a function of the temperature. Our sonification builds on basic acoustic properties of phase modulation. Only interesting structures like anti-/vortices remain heard, while everything else falls silent, without additional computational effort. The researcher interacts with the data by a graphical user interface. The sonification can be extended to any lattice model where locally turbulent structures are embedded in rather laminar fields.

1. INTRODUCTION

In our research, the usefulness of sonification for the display of numerical physics results is being studied.

One interesting model is the so-called *XY-model*. The model is programmed as a lattice of single spins, that may point in any direction in the 2-dimensional XY-plane. It will be explained in more detail below, but in this introduction, we want to point out why it is useful to sonify a 2-dimensional model, that may as well be visualized.

XY-models exhibit a special *topology*, i.e. we may find structures formed by the spins. These are vortices and anti-vortices. A vortex is defined as an arrangement of 4 spins (that we will refer to as spin quartet), which turns around $+2\pi$, if you follow it in counterclockwise direction. The anti-vortex turns by -2π (see Fig. 1). There is always the same amount of vortices and anti-vortices on a lattice with periodic boundary conditions.

We chose to sonify this model for various reasons. With the naked eye, the topological structures are hard to find (Fig. 2(b)). In a method called *cooling*, the overall energy of the configuration is lowered and only the most stable structures stay in an else laminar field (Fig. 2(c)). As a draw-

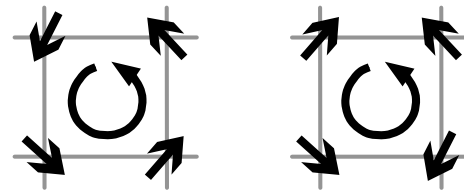


Figure 1: Scheme of an ideal vortex (left) and an anti-vortex (right). If one follows the spins in counterclockwise direction and adds up the angle differences, the vortex turns by $+2\pi$ and the anti-vortex by -2π .

back, cooling destroys all topological structures, if applied long enough. And at any step, physical information is lost.

Depending on the temperature as model parameter, the behavior of the vortices and anti-vortices changes. The exact changing point is called the Kosterlitz-Thouless phase transition. This cannot be *seen*, when observing the model, and may only be calculated with the help of nonlocal observables (measures of the whole configuration). A simple nonlocal observable is the vorticity, the number of vortices and anti-vortices in the configuration, but the phase transition can not be deduced by this analysis. Different sound properties at different states - below or above the phase transition - is what we wanted to achieve by utilizing sonification.

Finally it was a very interesting task to find appropriate, easy-to-hear sonifications for a problem that *is* visually displayable. The sonification has to make use of acoustic principles and thus leave visual thinking concepts behind. We believe, that this was achieved in the sonification we present in this paper. It can now also be extended to other models and higher dimensional data, where the hidden structures are still unknown and the data is not visualizable.

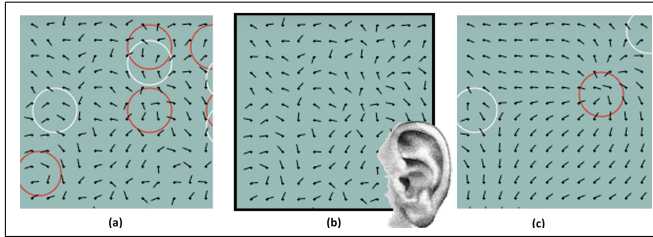


Figure 2: Detail of the XY-model. (a) shows a typical configuration, where also the positions of the vortices and anti-vortices have been calculated and are shown as red and white circles. If only raw data is shown (b), these structures are very hard to find visually. On the right hand side, (c) shows the same detail after several steps of cooling, an algorithm that lowers the overall energy and leaves only the most stable vortices and anti-vortices. In this kind of averaging procedure a lot of physical information is lost. Even the exact position of the structures changed during cooling. In our approach, we work only with the raw data (b). The sonification allows a quick overview over the state the system is in and detailed information on the position of the vortices and anti-vortices *without* calculating them.

2. SONIFICATION IN PHYSICS

In physics, many examples are known where sonification was used before. The most common cited are probably the Geiger counter or the Sonar. Physics is a huge and diversified field, and there are various sonification approaches, therefore an overview of the state of the art cannot be given in this short paper. Some approaches can be found in [1].

Many projects can be found at the border between science and arts, using sonification of physics data for exploratory and artistic reasons. For instance, algorithmic composition tools are based on physical event generation as the fission model (e.g., [2]) or scientific experiments become music (e.g. the piece *50 Particles*, [3]). In the AlloSphere, a 3-storey high sphere for virtual environments, also theoretical physics data shall be explored [4].

Two research projects ([5], [6]) have studied the sonification of numerical physics models, of which the current paper is a continuation.

3. SPIN MODELS - THE XY-MODEL

3.1. Spin models

Spin models are simple computational models, that use discrete or continuous data on a lattice (a discrete structure). Usually, the lattice forms a (hyper-) torus, i.e. one uses periodic boundary conditions. The degrees of freedom are called spins. In the simplest model, the Ising model, only 2 values are possible, spin up or spin down. A straight-

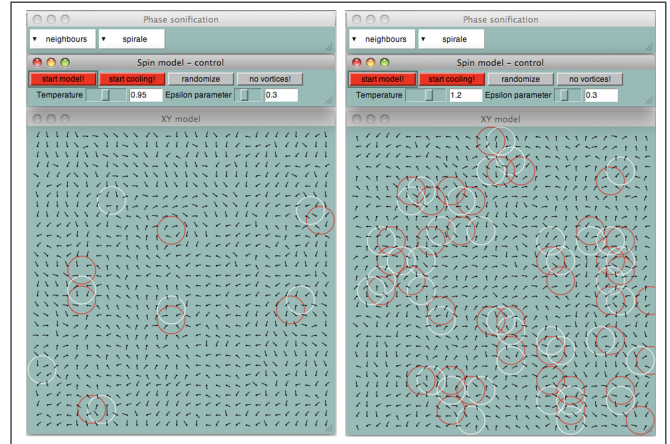


Figure 3: Visualization of typical configurations of the XY model below and above the phase transition (respectively on the left and right hand side). The higher the temperature, the more (red) vortices and (white) anti-vortices can be found. Vortices and anti-vortices usually form pairs, that appear more tightly bound above the phase transition.

forward extension is the XY-model, where the spin is continuous and may point in any direction in the plane.

The spins are linked to each other through an energy functional, depending on the temperature. The higher the temperature, the more the spins flip randomly due to heat induced fluctuations. The lower the temperature, the more they try to align with their neighbors. Between the low- and high-temperature *phases*, depending on the model, we find a phase transition at a critical temperature.

The Ising model describes, e.g., ferro-magnetic behavior (changing from the high-temperature non-magnetic to the magnetic phase). The XY-model is richer in structure. It exhibits topological objects, vortices and anti-vortices. They are defined as special configurations of the four spins located at the corners of an elementary square of the lattice (a plaquette). If the differences of the angles θ_i at the corners sum up to 2π when visiting them in the counter-clockwise direction, we speak of a vortex. For the case of -2π , an anti-vortex sits at the plaquette (compare Fig. 1). In case the sum is 0, no topological object is present at the plaquette. The structures are topologically stable and change their behavior depending on the temperature. The XY-model exhibits a Kosterlitz-Thouless phase transition [7]. At this transition, the vortex - anti-vortex pairs, that are close together at low temperatures, become unbound (compare Fig. 3).

3.2. Software implementation

Numerically, spin models can be treated with *Monte Carlo algorithms*. For each update of the spin configuration, a lat-

tice site is chosen, and a random candidate spin proposed. If the overall energy decreases, the new spin value is accepted. If the energy of the candidate configuration is higher, the new spin is accepted only conditionally based on a random decision. Else, the old spin value is kept. (For a description of the algorithm, see [8].)

In order to obtain smooth configurations of spins, we apply a *cooling algorithm*. In this case, the overall energy always decreases. The problem is, that cooling will average all vortex pairs out to establish a configuration of minimum energy, i.e. a laminar lattice with all spins aligned. Thus it depends on experience when to stop the cooling process. (See Fig. 2.)

We implemented the model and the sonification package (see below) in SuperCollider3, a free programming language developed for real-time audio synthesis [9]. A graphical user interface (GUI) allows to visualize the spins, and was used to produce the plots in this paper. The temperature can be changed dynamically. Also the location of vortices and anti-vortices can be computed, and is indicated in the GUI. The whole package runs well with a 38x38 lattice in real time, which is rather a small size for an up-to-date simulation of this system. Still, the behavior of the phase transition is correctly reproduced.

4. SPIN QUARTET SONIFICATION

For the sonification approach the plaquettes are the starting point – four neighboring sites on an elementary square, that carry the topological structures. We refer to them as *spin quartets*:

$$s_{x,y,i} = (s_i, s_{i+\hat{x}}, s_{i+\hat{x}+\hat{y}}, s_{i+\hat{y}}) \quad (1)$$

For each spin quartet, a sound grain y_r is played, where a sine oscillator is modulated depending on the spin values. The sonification operator¹ is given as:

$$\hat{y}_r(\hat{t}) = \sum_{r(R)} L_{n_s}^\infty \left[a_r(T, t_r) \cdot \mathcal{F}_{f_{0,r}}^{BRF,2} \left[y_r(\hat{t}, d_{x,y}, r) \right] \right] \quad (2)$$

In principle, the sonification signal $\hat{y}_r(\hat{t})$ is the sum over r spin quartets (within the range R). Each quartet is looped infinitely (or until the user changes the selection), $L_{n_s}^\infty$, over n_s samples. The signal of each spin quartet is $[y_r(\hat{t}, d_{x,y}, r)]$, as described below. It is filtered with a band reject filter \mathcal{F}^{BRF} of second order by a frequency $f_{0,r}$.

$$y_r(\hat{t}, d_{x,y}, r) = \sin(2\pi f_{0,r} \hat{t} + \phi_r(\hat{t}, d_{x,y})) \quad (3)$$

¹Recently, J. Rohrerhuber [10] suggested the formalization of the sonification operator, to make the mapping between the domain science and the sound synthesis more explicit. We take up this idea and extend the formalization by notation suggestions, as used in Eq. (2-5)

Each spin quartet data $d_{x,y}$ is used to modulate the phase of a sine oscillator.

$$a_r(R, t_r) = \text{Env}_{t_t, t_a, t_d, a_{max}(R)}[a] \quad (4)$$

The phase is constructed in the following way: the data values s' are cubically distorted and normalized. The distorted phase of an anti-/vortex still yields a final value of $\pm 2\pi$. Other configurations are suppressed, see Fig. 4. Then they are up-sampled by a factor of S (\uparrow_S) samples, and interpolated with a cosine function over $S/4$ samples.

The resulting phase distorted ramp is looped L_S over S samples. This is the phase that controls the phase of a sine oscillator with the base frequency f_0 . (This frequency is filtered out in the end, as described above, thus only the frequencies resulting from the phase modulation and their overtones remain in the signal.)

$$\phi_r(\hat{t}, d_{x,y}) = L_S \left[\uparrow_S \text{INT}_{\frac{S}{4}}^{\cos} \left[\frac{1}{4\pi^2} s_{x,y,i}^3 \right] \right] \quad (5)$$

The differences between adjacent spin values in the plaquette $\delta s_{x,y,i} = s_{x,y,i+1} - s_{x,y,i}$ are calculated assuring that the cumulation is continued in always the same rotational direction.

The $\delta s_{x,y,i}$ are added up in counter-clockwise direction to form a cumulative sum of the angles' differences $s'_{x,y,i}$:

$$s'_{x,y,i} = \sum_{n=1}^i s_{x,y,n} \quad \text{with } s_0 = 0 \quad (6)$$

For an ideal vortex and antivortex, $\delta s_{x,y,i}$ is $+\frac{\pi}{2}$ and $-\frac{\pi}{2}$. The cumulative sum $s'_{x,y,4}$ is accordingly $+2\pi$ and -2π . Neutral spin quartets containing no anti-/vortex² show a total rotation of $s'_{x,y,4} = 0$. (Any other configuration than anti-/vortices has values between $-\pi$ and $+\pi$, but with a total rotation of 0.)

The resulting curve s'_0 to s'_4 is used for the sonification shown in Eq. (6).

In the case of a vortex, the phase raises by 2π and the resulting frequency increases. In the case of an anti-vortex, the frequency is lowered due to a negative phase slope. The number of samples between each of the spins is $S = 30$. This results in a frequency of $f_p = r_s/4S = 44100/120 = 367.5 \text{ Hz}$. We choose a basic frequency f_0 of $3f_p = 1102.5 \text{ Hz}$. Thus, $(f_0 + f_p) = 2(f_0 - f_p)$, and a vortex and an anti-vortex are one octave separated.

The sonification is used interactively. Many spin quartets are played simultaneously around a central clicking point; their range of neighborhood, R , can be chosen, and $r = r(R)$ gives the number of simultaneously played quartets.

²This notation is used for 'vortex or anti-vortex'.

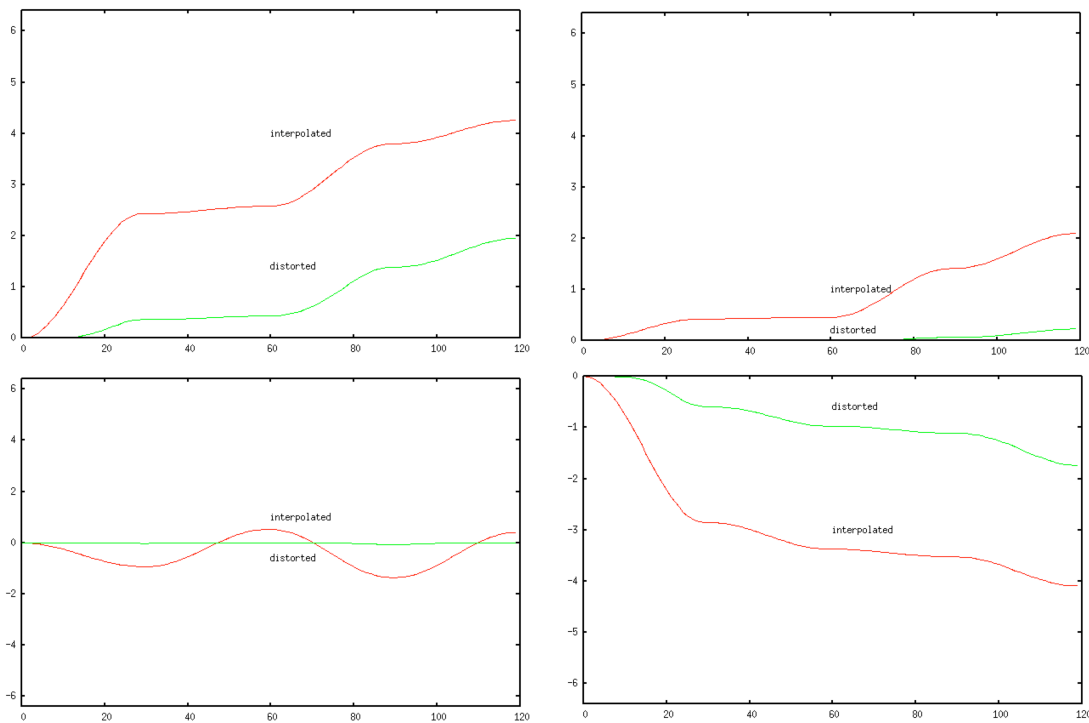
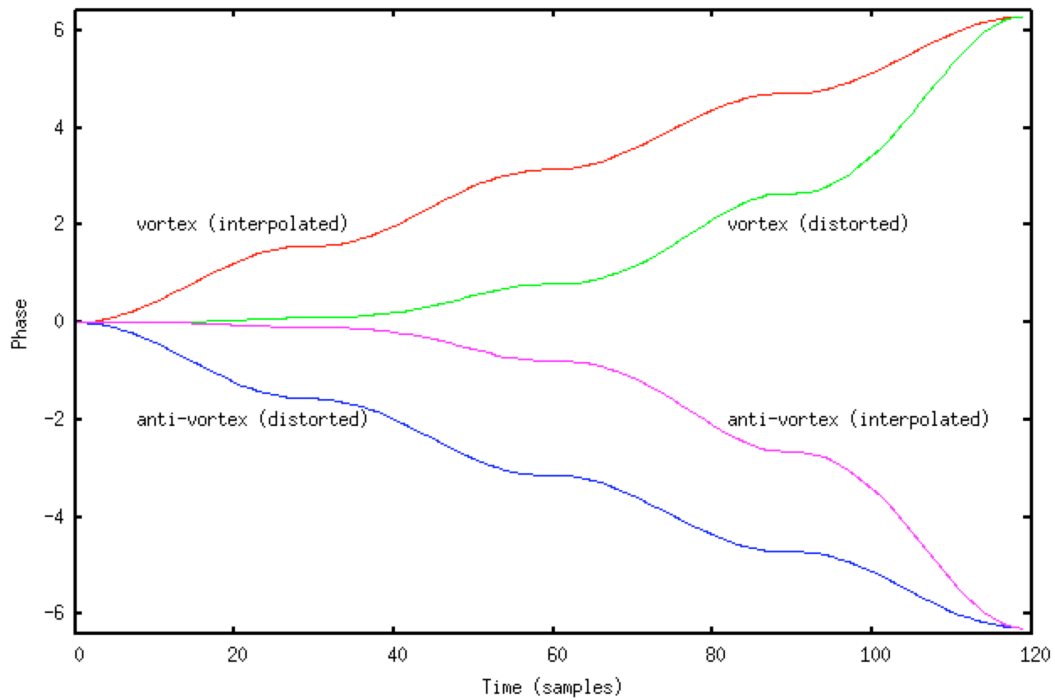


Figure 4: Phases of the ideal anti-/vortex (*upper figure*) and random configurations (*four lower figures*) for the XY sonification. The interpolated curves are depicted in red (and magenta), the distorted one according to Eq. 5 in green (and blue). The x-axis gives the number of samples. When the phase is looped, it is added smoothly to the last value, the base oscillation is periodic in 2π and does an effective modulo.)

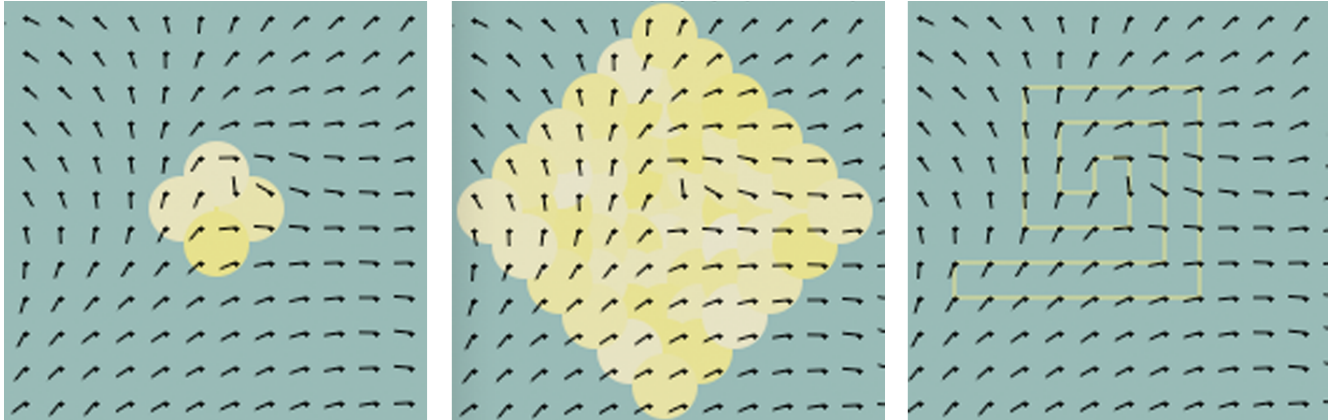


Figure 5: Interactive GUI of the XY model showing different listening range modes: In the neighbors mode (*left and medium figure*) a certain number of spin quartets around the clicking point is sonified. As an additional, more comprising mode of interaction, a spirale path was implemented. The spirale has variable length – order 1 gives a spin quartet. This setting is more exploratory, as the possible phase differences become more complicated. The sound of a vortex or anti-vortex depends on its position in the spirale and several anti/-vortices can be encompassed in one spirale.

A spotlight indicates all playing quartets in the GUI, see Fig. 5. Each sound is enclosed in an envelope Env and looped by the looping operator L , until a new site is chosen. The duration and loudness map the distance to the clicking point, thus closer neighbors sound louder and quicker, $a_r(t_r)$. This information is also encoded in a mistuning of the base frequency, thus $f_0 = f_{0,r}$. Very close anti/-vortex pairs will have nearly the same base frequency in octaves. If the pair is further shifted, the interval is mistuned, resulting in a beating of varying frequency. This is a key feature of the sonification that allows to distinguish the difference between bounded pairs and a vortex plasma. To give some orientation, left/right panning is applied, $\dot{y}(\dot{t}) = (\dot{y}_L(\dot{t}), \dot{y}_R(\dot{t}))$.

5. DISCUSSION

The presented sonification employs simple acoustic properties of a phase modulation. We believe that it is intuitive, as only the interesting parts sound and a laminar *field* of spins is silent. The difference between a vortex and an anti-vortex is absolutely clear, which is a major advantage over the visualization. The sonification gives also information where the eye has substantial problems of finding structures at all.

We did a short pre-evaluation. In a blind-test environment, the temperature had to be assessed in different settings: only-visual, only-audio and visual-plus-audio. The authors were the testing subjects. Even if the results are not statistically exploitable due to the small number of participants and test runs, a tendency was found that the audio-visual case had best results, shortly followed by the only-

audio case. The results of the visual-only case were much worse in terms of temperature assessment. Still, it has to be admitted, that the visual case was by far the fastest way of assessment. This shows that in a sonification, one often has to invest more time, but it also can lead to a more precise results.

6. CONCLUSIONS

In this paper we described a sonification of a system of statistical physics, the XY-model. Its interesting topology exhibits vortices and anti-vortices. These are defined by a nontrivial accumulated rotation of $\pm 2\pi$ and used to control the phase of a sine oscillator. The sonification allows to hear only the interesting structures, *without* calculating them beforehand. Vortices and anti-vortices can clearly be distinguished, even for untrained listeners. Information on the closeness of the vortices and anti-vortices is encoded by using slightly mistuned base frequencies, that result in a beating for remote vortices. The sonification is controlled interactively via a graphical user interface.

For the future, we will use the experiences gained with this model to study more complex ones. The ultimate goal is lattice QCD, a huge and high dimensional model, with a rich structure of topological objects.

Listening examples/screenshot videos.

Listening examples and further documentation can be found at <http://qcd-audio.at/results/xy>.

Acknowledgments.

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7. REFERENCES

- [1] K. Vogt, *Erstausgabe*. Karl-Franzens Universität Graz, 2008, vol. 1, ch. Sonification and particle physics.
- [2] S. Bokesoy, "Sonification of the fission model as an event generation system," *Proc. of the 7th Int. Conference on Digital Audio Effects (DAFX-04), Naples, Italy*, October 2004.
- [3] B. L. Sturm, "Composing for an ensemble of atoms: the metamorphosis of scientific experiment into music," *Organised Sound: Cambridge University Press.*, vol. 6, no. 2, pp. 131–145, 2001.
- [4] T. Höllerer, J. Kuchera-Morin, and X. Amatriain, "The allosphere: A large-scale immersive surround-view instrument," *IEEE MultiMedia*, vol. 16, no. 2, pp. 64–75, Apr.-June 2009.
- [5] SonEnvir-Team. Sonification environment research project. [Online]. Available: <http://sonenvir.at>
- [6] K. Vogt. Qcd-audio research project. [Online]. Available: www.qcd-audio.at
- [7] J. M. Kosterlitz and D. J. Thouless, "Ordering, metastability and phase transitions in two-dimensional systems," *J. Phys. C*, vol. 6, pp. 1181–1203, 1973.
- [8] H. Gould, J. Tobochnik, and W. Christian, *Introduction to Computer Simulation Methods*. Addison Wesley Pub Co Inc., 2002.
- [9] J. McCartney, "Rethinking the computer music language: Supercollider." *Computer Music Journal*, vol. 26, no. 4, pp. 61–68, 2002.
- [10] J. Rohrerhuber, "Sonification variables," in *Proceedings of the Supercollider Symposium*, 2010.