An Introduction to Sonification and its Application to Theoretical Physics

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Abstract: *During the last centuries, science has focused on visualisation. Current audio programming tools allow us to explore new ways of perceptualisation. Sonification is defined as the use of sound to perceptualise data and convey information. It is an alternative and complement to visualisation, and proved to be particularly useful in the analysis of highly complex and multi-dimensional data sets. In recent years, it emerged in different fields, pushed by increasing data amounts in science and the growing consciousness of the sense of audition. Potential benefits lie in the accurateness and multi-dimensionality of human audition.*
On the other hand, the computational approach to problems in physics has developed into a powerful method. Numerical simulation of systems plays a key role in statistical mechanics. The rapid development of computers and algorithms has led to new quantitative and qualitative insights. The analysis of huge and very complex data sets is required, and often only a few simple observables were considered.
Our interdisciplinary approach applies sonification techniques to numerical models of statistical physics. Some of these results were developed in the research project SonEnvir.

[Keywords: Sonification, Auditory Display, Audification, Computational Physics, Perceptualization, Spin Models]

1 AUDITORY DISPLAY

1.1 Background

During the last centuries, science focused on visualisation. Scientific data is usually presented in scientific graphs or more recently, animated graphics. A complementary approach to perceptualisation is auditory display, which has emerged in the last 20 years for a number of reasons. Firstly, data amounts have been increasing both in science and society due to growing CPU power. Thus new techniques of perceptualisation are being explored. Secondly, the study of music and with it the sense of audition under various scientific aspects became popular. Finally, audio synthesis has only reached sufficient quality and realtime efficiency in the last decades.
In 1992, the first International Conference on Auditory Display (ICAD) [20] took place in Santa Fe, following the initiative of Gregory Kramer. Its community consists today of a core of some 100 scientists, stemming from different disciplines, mostly psychology, computer science, sound engineering and music. Against this background, the interdisciplinary research project SonEnvir – Sonification Environment - took place in Graz from 2005-2007. Partner institutions from all four universities in Graz took part under the chair of the Institute of Electronic Music and Acoustics (IEM) of the University of Music and Dramatic Arts Graz. The Institute of Physics of the University of Graz collaborated with data stemming from computational models. This paper partly describes examples of sonification stemming from this collaboration.

1.2 Definitions

Auditory display (AD) is used in various aspects in science and technical applications. *Auditory graphs* allow the display of graphs for visually impaired persons [29]. Different kinds of sounds encode information more abstractly: *attensons* are sounds that draw our attention to a special circumstance (e.g., alarms); *earcons* are brief, structured sound patterns that represent a specific item or event; they are abstract but parameterisable encoding more information (e.g., Morse code); *auditory icons* are sounds recognizable from everyday life that stand for an item or event as well (e.g., the trash can sound of computer desktop systems). Both earcons and auditory icons are often used in computer-user interfaces.

In research, there are three standard tools of auditory display: audification, parameter mapping (sonification) and model-based sonification. Sonification is often used as the all-encompassing term. It is defined as the *use of non-speech audio to convey information.* [22]
Data is presented to the human senses by auditory display as a new, complementary approach. Simple examples for early auditory display devices are the Geiger counter and the Sonar.
Audification means the direct mapping of any data to a one-dimensional data stream that can be interpreted (and heard) as a sound wave (an example is given below in section 2.4).
Model-based sonification uses physical formalisms in order to explore high-dimensional data spaces [18]. *Sonification* in the narrow sense of parameter-mapping means a parameterisation of sound depending on properties of the data. Different aspects of sound are controlled by the data: location, loudness, pitch, melody, timbre and rhythm (see Figure 1). There, psycho-acoustic effects of auditory grouping play an important role, and in changing several of the above parameters, parallel audio streams can be synthesised.

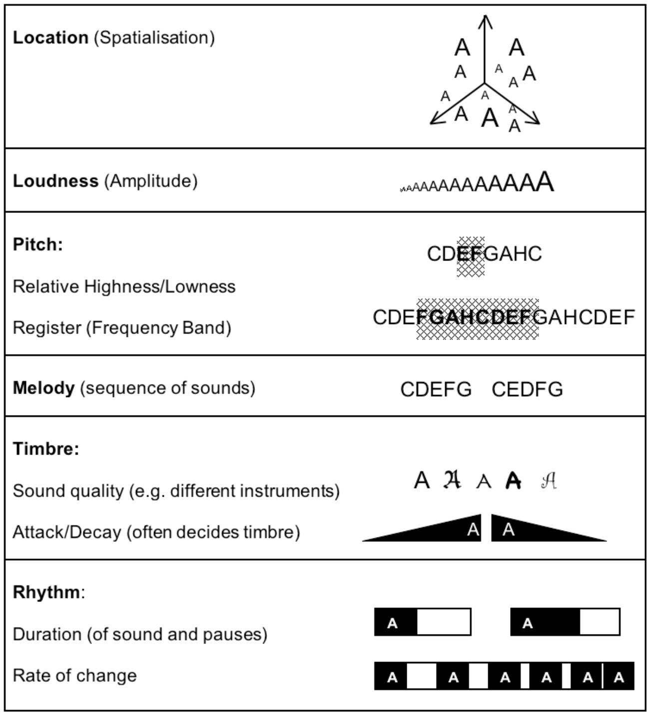


Figure 1 Parameters of sonification. (Source: [32])

1.3 Examples in Science

An early but illuminating example of research using sonification is the experiment of the inclined plane by Galileo Galilei. Following Drake [10], it seems plausible that Galilei used auditory information to verify the quadratic law of falling bodies. He attached little bells in quadratically increasing distance on an inclined plane, which were hit by balls rolling down; this resulted in a regular rhythm. Obviously, since then, science evolved into a rather visual science [9]. But also in modern science, sonification has already played a role: one example of audification is given in a paper by Pereverzev et al., where quantum oscillations between two weakly coupled reservoirs of superfluid helium 3 (predicted decades earlier) were found by listening to the amplified raw signal [25]. Other examples can be found in radio and plasma wave science in astronomy, where data of space missions is analysed in a first step as an audio signal (for an overview and examples see, e.g., [8]). Examples of sonification in other disciplines are, e.g., real-time monitoring of EEG data of epilepsy patients [6] and [1], studying oscillations of yeast cells with an atomic force microscope [23] or sonification in the social sciences [9].
In the examples cited above, the term sonification was not always used, as systematic research in this field is relatively recent.

1.4 Benefits and limitations

A strong argument in favour of sonification is the sophistication of human auditory perception. Audition is the highly developed human ability to extract information in different situations, both in everyday life and in professional contexts. From the sound of our footsteps we deduce the floor's consistency, car mechanics listen to the engine sound in order to reason about the causes of malfunction. Physicians utilise the stethoscope in auscultation for diagnostic purposes. The properties of the human auditory system make it an especially well-suited sense for the representation and exploratory analysis of - eventually multidimensional - data in the context of sonification.

The potential benefits of auditory displays are (see [19]):

- high *temporal and frequency resolution*, leading to high sensitivity to rhythm and pitch;
- the ability to listen to *more than one audio stream* in parallel;
- *eyes-free conditions*, which are perfectly suited for distance-monitoring;
- the formation of *auditory gestalt* (holistic listening) perceiving complex sound patterns as a whole;

- the ability either to draw attention to acoustic signals or assign low priority in alertness while remaining aware of the sound (*backgrounding*)

Limitations of auditory displays are:

- the *low resolution in some auditory variables* (e.g., brightness and spatial precision),
- the *lack of absolute value* and perceptual orthogonality,
- potentially, the *annoyance* of the sounds and their interference with verbal communication
- *no persistence* in auditory display since time is an essential element in sonifications, unlike a print-out.
- human capabilities to interpret sound have to be trained, which might be difficult in some cases (*user limitations and cultural bias*)

Auditory display can be as effective as visual display - the audio-visual combination produces further benefits for complex information display [1]. Auditory displays can be utilized to enhance graphical user interfaces (e.g., SonicFinder [15]) or substitute them for visually impaired people ([11], [12], [13] and [14]).

Listening skills can be trained - e.g. Speeth [28] showed in 1961 that subjects were able to classify bomb blasts and earthquakes in the audification of seismic data.

Systematic approaches to designing auditory displays have to take into account a concise analysis of both, the tasks to be performed by the user, and the data structure. Appropriate data pre-processing can be a helpful prerequisite for successful sonifications.

There is a cultural bias towards visualisation because methods to store, represent and manipulate graphics have a longer history than those for sound. Furthermore, we are able to integrate hypothetical interpretations in visual form into a graphical display (trend lines, groupings etc.) and it is easier to communicate about a graph, because we can simply point to a specific element. Visualisation techniques and our learnt understanding of them have been refined since the very beginnings of modern science itself. Compared to visual display, scientists are not trained to work with auditory display, so they first have to learn how to interpret sound before a sonification will show full benefits.

1.5 Aims of sonifications

(a) Alternative representation of data.

Auditory graphs and respresentation of multi-dimensional data can be used for didactic purposes, help visually impaired people and create multi-modal displays.

(b) Use of audition in order to analyse scientific data.

Several examples of sonification in research are given above (section 1.3). Of course, sonification can only be a complementary tool to classical analytical methods, but it may be a crucial one. We accept, for instance, visual interpretation in many scientific fields as an analysis tool, which is often superior to or preceding mathematical treatment. For instance, Marsaglia [23] described tests for the quality of numerical random number generators. One of these is the parking lot test, where mappings of randomly filled arrays in one plane are plotted and visually searched for regularities. In the description, he argues that *visual tests are striking*, but *not feasible in higher dimensions.* An all-encompassing mathematical or numerical test of this task cannot be provided, as one does not know beforehand (and should not make too many assumptions) which kinds of patterns to expect. Sonification is a logical continuation of such analytical methods.

(c) Artistic applications.

Examples of music using sonification are pieces composed for the ICAD concerts and SonEnvir. Two examples shall be mentioned here as examples: During SonEnvir, Till Bovermann and Jonas Groten elaborated a performance using interactive sonification of a juggling show with the help of a tracking system at the IEM [3]. A composition based on socio-economic data as requested by the ICAD concert board in 2006 is ‘Navegar É Preciso’ by Alberto de Campo and Christian Dayé [3].

1.6 Methodology

Sonification is usually done with audio synthesis software. We work with SuperCollider3 (SC3) [26]. This is an object-oriented language and programming environment under the free GPL license. It was originally developed by James McCartney as a real time sound synthesis language, and is continually enhanced by a growing community of users in the electronic music domain and the sonification community.

Data usually has to be pre-processed. As the sonifications are adapted to each new problem, typical data sets are used to test them. For data generation, we use SC3 for the smaller models as well, as the program settings can easily be changed interactively. For bigger models or pre-recorded data we use programs in Fortran and/or data of the Institute of Physics of the University Graz.

As aural spatialisation is an important acoustic parameter, we use the CUBE, a multi-channel performance hall at the IEM. For better dissemination of the results we render versions of sonifications in stereo.

Usually, sonification tools are designed in a way to allow interactivity via different interfaces. This enhances the effectiveness of the analysis, e.g. by scrolling quickly through data and zooming in to an interesting subset, where also sound parameters can be adapted to reveal special patterns best. Also long-time attention and didactic usability might be enhanced by interactive tools. [17]

2 SONIFICATION OF SPIN MODELS

2.1 Introduction

In the course of the SonEnvir project, we sonified spin models of statistical mechanics. Other approaches within the SonEnvir project dealt with Baryon spectra of Constituent Quark Models [4]. Other applications in the field were simple sonifications of the Dirac spectrum [5]. Smaller projects dealt with the chaotic double pendulum and the logistic equation (see [27]). All these examples and the sound files of the sonifications below can be found on the SonEnvir project homepage http://sonenvir.at/data.

2.2 Special role of sonification in physics

In physics and related disciplines, sonification in the narrow sense of parameter mapping has special advantages. In principle, there are more parameter dimensions available than in visualisation (see Figure 1). Particle physics, e.g., is usually described in a 4d framework. This makes it hard to visualise and thus very abstract. (There are of course attempts to visualise high dimensional data, e.g., in quantum mechanics [29].) Even if we handle a 3d space evolving in time, a complete visualisation is not possible any more.

Audio streams are not orthogonal (like coordinate axes in a graph) but can rather be compared to mathematical subspaces, a common concept in science. Furthermore, many phenomena in nature are wave phenomena happening in time, just as sound is. Thus sonification provides a very direct mapping. While scientific graphs often map physical phenomena as a function of time, this is not necessarily the case in a sonification, where the physical time persists, and more parameters may be displayed in parallel.

A disadvantage of the sonification of physical data is that exact quantification is not possible. Whereas numbers can be read out (with some degree of accuracy) from a graph, the exact pitch or loudness can usually not be told from an auditory display.

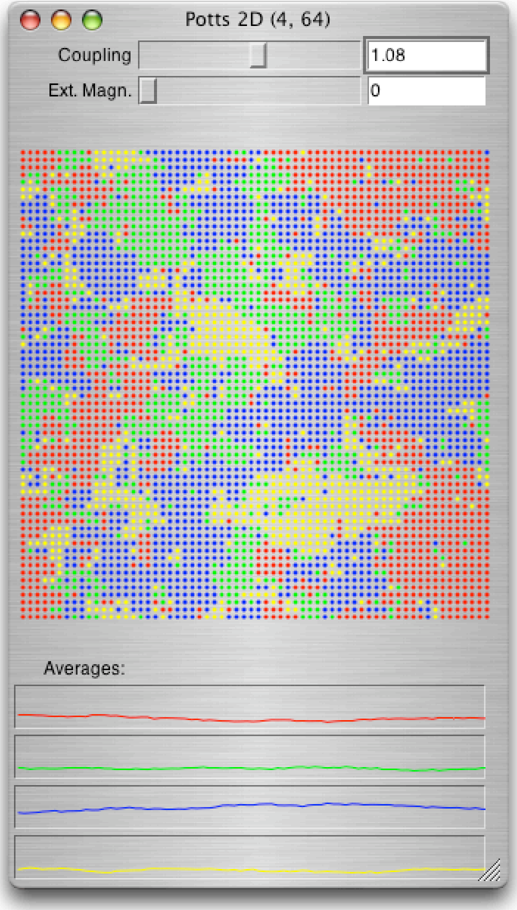
On the other hand, qualitative analysis, pattern recognition and dynamic displays can sometimes be better done with sound.

2.3 Physics background

Spin models describe macroscopic properties of materials (e.g., ferro-magnetism) by simple microscopic interactions between single elements of the material. They are nowadays computed in simulations. The best-known and well-understood examples are the Ising and the Potts model. Their Hamiltonian, giving the overall energy, is given by

$$H = J\sum_{i,j} S_i S_j - M\sum_{i} S_i \tag{1}$$

where J is the coupling parameter between spin S_i and its neighbouring spin S_j. J is inversely proportional to the temperature; M is the field strength of an exterior magnetic field acting on each spin S_i. The first sum is denoted over nearest neighbours and describes the coupling term. It is responsible for the phase transition. If J=0, only the second term remains, and the Hamiltonian describes a paramagnet, being only magnetised in the presence of an exterior magnetic field. In our simulations, M was always 0.



Potts 2D (4, 64)

Coupling 1.08

Ext. Magn. 0

Figure 2 Graphical User Interface of a Potts Model with 4 different spin states. Mean magnetisations are given below

The physical interpretation is, that the magnet consists of simple atoms on a quadratic (or in three dimensions cubic) lattice. At each lattice point an atom is located with a magnetic moment (a spin) up or down. In the computation, on the one hand, neighbouring spins try to align to each other, which is energetically more favourable. On the other hand, an overall temperature causes random spin flips. At a critical temperature T_{crit}, this process is undecided and there are clusters of spins on all orders of magnitude. If the temperature is lowered from T_{crit}, one spin orientation will prevail. (Which one is decided by the random initial setting.) Macroscopically, this is the magnetic phase ($T< T_{crit}$). At $T> T_{crit}$, the thermal fluctuations are too strong for uniform clusterings of spins. There is no macroscopic magnetisation, only thermal noise.

2.4 Sonifications of discrete spin systems

Spin models are usually exploited by abstraction, but details are often suppressed and an intuitive understanding is not attained. The standard approach often aims at a quantitative and visual exploitation of results, for instance by reducing the dimensions (e.g. a multi-dimensional system can be mapped into one plane).

Information hidden in the statistical fluctuations of the Ising and Potts model are its current phase (e.g., magnetised or non-magnetised) and the order of its phase transition (continuous for $q \leq 4$ states or first order for $q \geq 5$ states). (For further information see [33].)

The sonifications allow differentiating between the phases. Special emphasis was laid on 3d lattices that are evolving dynamically. We also studied different orders of phase transitions in the Potts model, and our sonification allows at least for a first estimation of the order, even for untrained listeners. Details are given below or in [31] and http://sonenvir.at/data/spinmodels/, where also audio files can be downloaded.

(a) Audification

Our first attempt was an audification. For an audification, a mapping of the data into one dimension is needed, a so-called sequentialisation. This can be done on different paths. We found the Hilbert curve, a space filling curve for quadratic structures, especially as neighbourhood relationships are preserved reasonably well [16] (Figure 3).

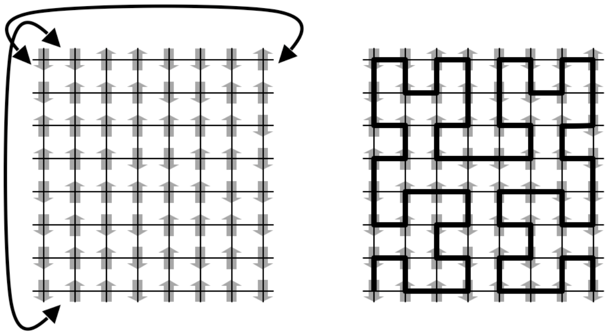


Figure 3 Scheme of two different sequentialisations. Left side: a *torus path*, where the grid is read out line by line and column per column. Right side: *Hilbert curve* (space filling Peano curve for quadratic structures).

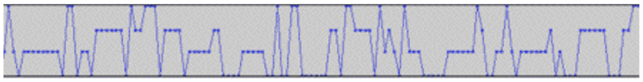


Figure 4 Resulting audio wave of sequentialised 4-state Potts model (first 23 ms).

This signal is used to modulate the frequency of a simple sine wave (thus, strictly speaking, it is not a pure audification) or taken directly. The resulting sound is noise in the high temperature phase. Below it, clusters of equal spins emerge, resulting in a rough sound quality.

(b) Parameter Mapping

For a 3d Ising model evolving in real-time, we developed a parameter mapping sonification, also to allow for a more pleasant soundscape. 3 sound parameters (pitch, noisiness and spatial location) are used to manipulate the character of a sound grain, a short sound event. When many of these sound grains are played in a short time frame, a *gestalt* of the whole lattice is perceived.

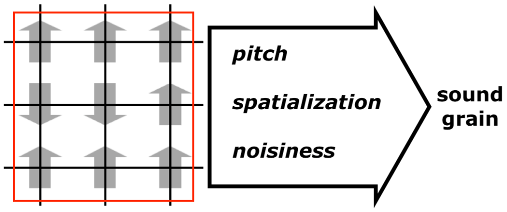


Figure 5 Scheme of the applied parameter mapping.

The data pre-procession and parameter mapping is done as follows: An average value over 3^d spins is taken, where d is the dimension of the lattice. This value determines pitch and noisiness of the soundgrain – the more spins are alike, the purer and higher/lower is the tone. In 3d, the location of the region is mapped in a virtual space, which requires a multi-channel system.

This sonification allows a more pleasant sound, and still even untrained listeners can distinguish the phases.

(c) Separated Channels

Finally, we studied the order of the phase transition and whether the difference between first order and continuous transitions can be heard. Data for configurations with continuously falling temperatures was simulated and stored. The sonification is a refined audification for separated spin orientations, played on different audio channels and allows for a first categorisation.

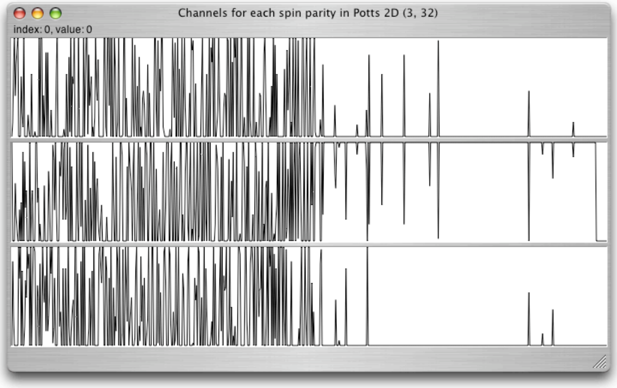


Figure 6 Parallel audifications of all 3 states in a 3-state Potts model. The temperature is continuously decreased. The critical temperature is seen in the middle of the simulation.

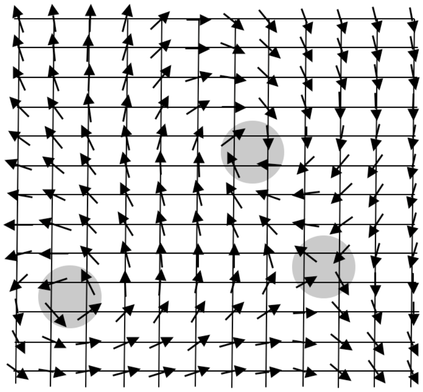


Figure 7 Schematic configuration of the XY-model with stable vortex-anti-vortex pairs (indicated with grey circles). These are so-called topological structures searched for.

3 CURRENT WORK – MULTIDIMENSIONAL AND CONTINUOUS DATA

Currently, we work on generalising the type of models that can be studied with our sonifications. Two extensions are of importance in physics: firstly, continuous models (rather than the discrete Ising and Potts model), secondly models of more than 3 dimensions (typically 4d in QCD).

3.1 The XY model

An interesting continuously valued spin model is the *planar XY model*, which exhibits no phase transition of second order but a Kosterlitz-Thouless (KT) transition [21]. The spins may assume values on the unit circle and the model has a continuous U(1) symmetry. Topological structures can emerge, so-called vortices and anti-vortices. The KT transition is driven by a melting of vortex-anti-vortex pairs. This mechanism is of topological nature and is very different from the long-range ordering of spins which causes the transition in the discrete models. There is no such long range order in the 2d XY model.

The XY model is an ideal further study object for sonification. Data is continuous and of non-trivial topology. Vortices are spin waves, and an audification attempt seems promising for displaying the difference in the phases. These are not apparent in a visualisation. Currently, we are applying our sonifications to this problem.

3.2 Future Work

Multi-dimensional data sets, stemming from quantum chromodynamics (QCD) will be our ultimate test case. A short pilot study was done in collaboration with T. Bovermann from the Neuro-informatics Group of the University of Bielefeld. Data was (and will be) provided by the QCD (quantum chromodynamics) lattice group of Prof. Chr. Gattringer and Prof. C. B. Lang from the University of Graz. This data stems from current simulations describing the behaviour of the most elementary particles known today. The challenge is that the data sets are 4-d and huge, thus methods have to be developed that allow a quick overview over the data.

4 CONCLUSION

This paper describes a rather new method of scientific data exploration, the use of auditory display. While similar principles using human audition for research have been applied throughout many years (even centuries), the term sonification, and systematic research in this field, are only about 20 years old.

In comparison to the visual sense, human audition is very sensitive to dynamic changes and covers a wide range of perception, both in frequency and amplitude. Researchers may benefit from this highly sensitive measure as they evaluate results out of scientific graphs.

During the research project SonEnvir, sonifications for many different scientific disciplines were developed – also for physics. Data stemming from numerical models of statistical physics was sonified in different ways. While *audification* allows a rough sounding scan through all data, *parameter mapping* makes use of the multi-dimensionality of the human ear. In our example, spatialisation, pitch and timbre of the sound are controlled by the data. This line of research is continued by focusing on continuous and higher dimensional data.

The biggest problem of sonification is, that scientists are still not used to it, and that visualisation techniques have been refined and learnt for centuries, while auditory display is still at the beginning. We are confident that new perceptualisation methods help finding new points of view and thus new paradigms in science.

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